

LCR2D: 2D field solver for calculation of inductances, capacitances and resistances

LCR2D was developed by M. Khapaev,
Moscow State University,
Faculty of Computer Science,
Dep. of Numerical Methods.
vmhap@cs.msu.su, mkhap@pn.sinp.msu.ru

April 24, 2005

Contents

1	Introduction	2
1.1	What LCR2D is Intended for?	2
1.2	Why This Program Was Written?	2
1.3	LCR2D Essentials	3
1.4	Hardware and Software Requirements	3
1.5	Setup Procedure	3
1.6	How to Start the Calculations and Obtain the Results	3
2	Problems Definitions	4
2.1	Basic Problem: Matrix of Self and Mutual Capacitances ($\mathbf{C}$) . . .	4
2.2	Matrix of Self and Mutual Inductances ($\mathbf{L}$)	5
2.3	Matrices of Thin Film Ohmic Conductances ($\mathbf{G}$) and Resistances ($\mathbf{R}$)	6
2.4	Thin Superconductive Films Terminal-to-Terminal London In- ductances	7
3	Numerical Technique	8
3.1	The Potential Representation of the Solution	8
3.2	The Numerical Algorithm	8
4	The Input File	8
4.1	An Example	8
4.2	The Input File Structure	10
4.3	Input Data in General	11
4.4	Understanding the Boundary Elements	11
4.5	Input Data Definitions	12
5	The Example of Calculations	15
6	Effective Calculations	16
7	Possible Problems and Error Messages	16

1 Introduction

1.1 What LCR2D is Intended for?

The program **LCR2D** has been developed as general, fast and reliable tool for the extraction of matrices of self and mutual capacitance $\mathbf{C}$, inductance $\mathbf{L}$, conductance and resistance $\mathbf{G}$ and $\mathbf{R}$ by use of the 2D approach. The program can be implemented for:

- The extraction of the parameters of long multi-conductor transmission lines buried in multilayered media;
- Calculation of matrix of conductance of multiport thin film resistors;
- Estimation of terminal-to terminal superconductive thin film kinetic inductances;
- Calculation of thermal conductivity parameters.

The program **LCR2D** is fast and precious. It can be used in CAD applications for the extraction of the models parameters. Probably, in many cases our program can be fast enough for direct calculations as the subroutine in the process of circuit optimization.

1.2 Why This Program Was Written?

All problems the program **LCR2D** can solve in general can be reduced to the problem of extraction of the matrix of capacitances of a multi-conductor transmission line buried in non-homogeneous media. This problem was of very high interest in microelectronic and other areas. The considerable attention was paid during last decades for the development of the computer programs for this problem [1, 2, 3, 4, 5, 15, 7, 8, 9, 10]. The general and the detailed survey of the state of the art in this area can be found in [11].

Most of the works concerned capacitance extraction are based on the nearly similar technique, "moments method" or indirect Boundary Element Method (BEM) [12]. The other works use finite element method (FEM) (for example, [9]) or other technique closed to BEM or FEM. By many reasons BEM is more preferable and leads to more general and effective programs [11], [9].

Nevertheless, the conventional version of method of moments (BEM) is not effective enough and can result in long and inaccurate calculations. The reason is in the significant influence of corner points, proximity effects, different boundary conditions interfaces and poor convergence of the moments method. As the result, the program realization of the conventional method of moments does not realize all the advantages of the boundary element technique.

The program **LCR2D** is also based on indirect boundary element method. This program exploit another general scheme of BEM [13] which is different from the conventional method of moments and is more adequate the problem. Another improvement in the numerical technique is the effective mesh grading approach [14] with the enhancements partly presented in [15].

As result, **LCR2D** realize the convergence $O(h^2)$ where h is the parameter of mesh density. For conventional moments method the first or worse order of convergence is natural [14], [15].

The other improvement is the iteration solver GMRES [16], included in the program. Due to it and disk memory usage even very large problems can be solved on the ordinary old PC.

The improvements in the numerical technique represented in **LCR2D** results in the speed and accuracy of calculations as well as in the generality of the program.

1.3 LCR2D Essentials

The features of the program **LCR2D** are:

- Arbitrary form of the conductors, including both infinitely thin and thick;
- Arbitrary form of the interfaces between different layers of medium including conformal dielectrics;
- Arbitrary mutual location of conductors and interfaces;
- Realistically curved forms of domains without polygonal approximation;
- Correct treatment of unbounded domains by applying null total charge condition;
- Efficient treatment of corners, side-proximity effects and boundary conditions of different types;
- The program contain Green function of free space, Green functions for one and two parallel ground planes, two-layer medium Green function [5] and some others;
- The internal library of Green functions and basic curvilinear boundary curves can be easily extended;
- Optional out-of-core iterative GMRES-type solver;
- High accuracy and speed of calculations;

1.4 Hardware and Software Requirements

The current version of **LCR2D** is written for W32.

Console version of the program **LCR2D** is written using G77/GCC.

1.5 Setup Procedure

You need to copy the executable file and examples.

1.6 How to Start the Calculations and Obtain the Results

Print in command line:

```
LCR2D inputfilename [ outputfilename ]
```

The file *inputfilename* is text file and can be prepared with text editor. Parameter *outputfilename* is optional. Except the screen output, after the calculations two output files will appear, *TRACE.LCR* and *RESULT.LCR*. File *RESULT.LCR* is default for *outputfilename*. File *TRACE.LCR* contain control output of input data. In both files, the result of calculations can be found.

2 Problems Definitions

2.1 Basic Problem: Matrix of Self and Mutual Capacitances ($\mathbf{C}$)

Consider the problem of calculation of per-unit length matrix of electrostatic capacitance C of a stretched multiconductor transmission line, embedded in piece-wise constant non-homogeneous or multilayer media. The number of conductors in the problem is $M + 1$ if infinite ground planes are not present. In this case, the conductor with number 0 is considered as grounded. If the problem contain infinite ground plane(s), then the number of conductors *can be* M ($M + 1$ conductor 0 conductor grounded are also is admitted). Conductors (cross-sections) can be of arbitrary form. Multi-connected conductors are possible, so unique conductor can be presented by several transmission lines. Conductors can be thick and/or infinitely thin.

We assume the problem is 2D problem and electrostatic potential is $\varphi = \varphi(r)$, $r = (x, y)$. Then the problem is formulated as the boundary value problem for the elliptic equation for potential $\varphi(r)$, $r \in D$, D is the domain filled by dielectric media and external to all conductors:

$$\nabla(\varepsilon(r)\nabla\varphi(r)) = 0. \quad (1)$$

Relative dielectric permittivity $\varepsilon(r)$ is assumed to be piecewise-constant. Domain D can be bounded or unbounded. If it is unbounded we assume all dielectric-dielectric interface boundaries are bounded. If D is unbounded and ground planes are not present, the boundary condition on the infinity is posed:

$$\varphi(r) \rightarrow \text{const} \quad \text{if} \quad r \rightarrow \infty. \quad (2)$$

Boundary conditions on the boundary ∂D , the surface of the conductors, are:

$$\varphi(r) = V_i, \quad i = 1, \dots, M; \quad (3)$$

$$\varphi(r) = 0, \quad i = 0; \quad (4)$$

On the remote boundary or surface of symmetry (reflection)

$$\varepsilon(r) \frac{\partial \varphi(r)}{\partial n} = 0, \quad (5)$$

where n is the normal to boundary of symmetry. On curves of discontinuity of $\varepsilon(r)$ (interface boundaries) we set the interface conditions

$$\varphi_+(r) = \varphi_-(r), \quad (6)$$

$$\left(\varepsilon(r) \frac{\partial \varphi(r)}{\partial n} \right)_+ = \left(\varepsilon(r) \frac{\partial \varphi(r)}{\partial n} \right)_-, \quad (7)$$

where indexes $+$ and $-$ means the limiting values of the functions from different sides of the boundaries.

The total charge induced on the ("thick") conductor i with the boundary C_i is:

$$\sigma_i = \varepsilon_0 \int_{C_i} \varepsilon(z) \left(\frac{\partial \varphi(r)}{\partial n} \right)_- ds_r, \quad (8)$$

As the problem is linear, the vector of conductor charges $\vec{Q}$ and the vector of conductor potentials $\vec{V}$ are coupled by $M \times M$ capacitance matrix C :

$$\vec{Q} = C\vec{V}. \quad (9)$$

The capacitance matrix C is symmetric and positive definite.

Our main task is to calculate the matrix C . The calculation is performed on the base of (9). Setting V_i as 0 and 1, the calculation of the elements of C are carrying out successfully.

2.2 Matrix of Self and Mutual Inductances ($\mathbf{L}$)

Consider the problem of extraction of per-unit length matrix of inductances L of a stretched multiconductor transmission line, embedded in multilayer (or non-homogeneous) media with sectionally constant relative magnetic permittivity μ . We assume the problem can contain ground planes, carrying the return current. The number of conductors in the problem is $M + 1$ if infinite ground planes are not present. In this case, the conductor with number 0 is considered as carrying all return current. If the problem contain infinite ground plane(s), then the number of conductors *can be* M ($M + 1$ conductors, 0 conductor grounded are also is admitted). Conductors cross-sections can be of arbitrary form (for example, multi-connected).

It is assumed that the main boundary condition is the null normal component of magnetic inductance $\vec{B}_n = 0$ on the surface of the conductors and on ground planes.

We also assume that currents in the conductors are surface currents and has the sheet density $\vec{j}$.

Consider the magnetostatic equations for magnetic inductance $\vec{B}$, magnetic field $\vec{H}$, current $\vec{j}$ and vector potential $\vec{A}$:

$$\nabla \times \vec{H} = \vec{j}, \quad (10)$$

$$\vec{B} = \mu_0 \mu \vec{H}, \quad (11)$$

$$\vec{B} = \nabla \vec{A} \quad (12)$$

The conditions on the interfaces boundaries are:

$$\left(\vec{B}_n\right)_+ = \left(\vec{B}_n\right)_-, \quad (13)$$

$$\left(\vec{n} \times \vec{H}\right)_+ = \left(\vec{n} \times \vec{H}\right)_-,$$

where $\vec{n}$ is the normal to the interface, values with $+$ and $-$ indexes are the left and right limiting values. Consider the equation for the vector potential:

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \vec{A}\right) = \mu_0 \vec{j}. \quad (14)$$

We assume our problem is the 2D problem and $r = (x, y)$, $\vec{B} = (B_x(r), B_y(r), 0)$, $\vec{H} = (H_x(r), H_y(r), 0)$, $\vec{j} = (0, 0, j_z(r))$, $\vec{A} = (0, 0, A_z(r))$. Then the equation (14) is reduced to

$$\nabla \left(\frac{1}{\mu(r)} \nabla A_z(r)\right) = 0. \quad (15)$$

written in the domain D external to all conductors. The interface conditions on the boundaries of discontinuity of $\mu(r)$ in the embedding media are reduced to

$$(A_z(r))_+ = (A_z(r))_-, \quad (16)$$

$$\left(\frac{1}{\mu(r)} \frac{\partial A_z(r)}{\partial n} \right)_+ = \left(\frac{1}{\mu(r)} \frac{\partial A_z(r)}{\partial n} \right)_- \quad (17)$$

The boundary condition $\vec{B}_n = 0$ on the conductor cross-section boundary brings us to

$$A_{z,i}(r) = A_i \equiv \text{const}, \quad (18)$$

where i is the number of conductor. For grounded conductor $i = 0$

$$A_{z,0}(r) = 0. \quad (19)$$

On ground planes $A_z(r) \equiv 0$. Values of $A_{z,k}$ are also per-unit-length flux of magnetic field, associated with the conductor k [1].

The total current flowing in the k -th conductor with the boundary C_k is

$$I_k = \int_{C_k} j_z(r) dl, \quad k = 0, 1, \dots, M, \quad (20)$$

$$\sum_{k=0}^M I_k = 0. \quad (21)$$

From (10) and Stoke's theorem follows that

$$\mu_0 I_k = \int_{C_k} \left((\vec{H})_-, d\vec{l}_{C_k} \right) = \int_{C_k} \left(\frac{1}{\mu(r)} \frac{\partial A_z(r)}{\partial n} \right)_- dl. \quad (22)$$

Consider vectors of length M of total currents $\vec{I} = (I_1, \dots, I_M)$ and the vector of potential values $\vec{\Phi} = (A_{z,1}, \dots, A_{z,M})$. As the problem is linear, exists $M \times M$ matrix of inductances L

$$L\vec{I} = \vec{\Phi} \quad (23)$$

The inverted matrix L^{-1} can be found in the way similar to capacitance matrix C with dielectric permittivities $\varepsilon(r) = 1/\mu(r)$.

2.3 Matrices of Thin Film Ohmic Conductances ($\mathbf{G}$) and Resistances ($\mathbf{R}$)

Consider conducting planar structure with piecewise-constant sheet conductivity $\sigma(r)$. The ohmic current is flowing over the conducting plate due to the voltage drops between the terminals. The number of terminals is $M + 1$. The terminal with number 0 is considered as "reference" terminal. If the problem contain infinite ground plane terminal, then the total number of terminals can be M or $M + 1$.

Potential distribution $\varphi(r)$, $r = (x, y)$ satisfies the Laplace equation

$$\nabla(\sigma(r)\nabla\varphi(r)) = 0. \quad (24)$$

The Ohmic electrical current distribution in the plate is given by

$$\vec{j} = -\sigma(r)\nabla\varphi(r). \quad (25)$$

The boundary conditions for Laplace equation for terminals C_i are:

$$\varphi(r) = V_i, \quad r \in C_i, \quad i = 1, \dots, M, \quad \varphi(r) = 0, \quad r \in C_0. \quad (26)$$

The interface conditions on the curves of the discontinuity of $\sigma(r)$ are the conditions of the continuity of the normal current:

$$(j_n)_+ = (j_n)_-. \quad (27)$$

For the walls,

$$\frac{\partial \varphi(r)}{\partial n} = 0. \quad (28)$$

The total currents through terminals are:

$$I_k = -\sigma \int_{C_k} \frac{\partial \varphi(r)}{\partial n} dl. \quad (29)$$

Consider vectors of total currents through terminals $\vec{I} = (I_1, \dots, I_M)$ and vector of impressed voltages $\vec{V} = (V_1, \dots, V_M)$. The problem is linear and conductance matrix G is the matrix in the relation

$$\vec{I} = G\vec{V}. \quad (30)$$

This model (and our program) can be applied for the simulation of the contact resistance effects in resistor complexes [6].

2.4 Thin Superconductive Films Terminal-to-Terminal London Inductances

The thin planar superconductive multi-terminal film structure is considered. The stationary distribution of sheet superconductive current $\vec{j}(r)$, $r = (x, y)$ in the film due to Josephson phase difference between the terminals is can be reduced to the London equations for thin films:

$$\mu_0 \lambda_{\perp} \vec{j}(r) + \vec{A}(r) = \nabla \chi(r), \quad \nabla^2 \chi(r) = 0. \quad (31)$$

In (31), $\vec{A}(r)$ is vector potential of current $\vec{j}(r)$ and function $\chi(r)$ is the Josephson phase. Parameter $\lambda_{\perp}$ in (31) is

$$\lambda_{\perp} = \frac{\lambda^2}{d}, \quad (32)$$

where λ is London penetration depth and d is the thickness of the film.

The problem is really 3D problem due to vector potential term in (31). In some cases this term can be omitted and we obtain the next equation for the superconductive current

$$\mu_0 \lambda_{\perp} \vec{j}(r) = \nabla \chi(r), \quad \nabla^2 \chi(r) = 0. \quad (33)$$

The equations (33) are similar to Ohm law with the conductivity $\sigma = -(\mu_0 \lambda_{\perp})^{-1}$. This approach is very rough and is capable to estimate only the "kinetic" part of inductances matrix but it is more accurate than simple estimations on the base of "number of squares" method.

The problem is similar now to the problem of thin film Ohmic conductance matrix extraction. Instead of conductances matrix the admittance matrix Y will be calculated and the inductances matrix L is equal to Y^{-1} .

3 Numerical Technique

3.1 The Potential Representation of the Solution

In the program, single layer potential representation of the solution of (1), or indirect Boundary Element Method (BEM) is used. The main features of this technique can be found in [12] and [13]. The solution of the boundary value problem is presented as

$$\varphi(r_0) = \int_{\Gamma} G(r, r_0) \sigma(r) dl_{\Gamma} + \omega, \quad (34)$$

where integration curve Γ include the boundaries of conductors, interfaces boundaries and external boundaries of the domain but does not include boundaries with conditions are taken into account by Green function $G(r, r_0)$. The function $\sigma(r)$ is related to the induced charge densities. Furthermore, the kernel $G(r, r_0)$ in (34) is the Green function of free space ($-\log|r - r_0|$) or a Green function for certain boundary conditions: multilayer media, infinite ground planes and boundaries of symmetry. The value of the constant ω can be taken 0 for bounded domain D and need to be taken as 0 for unbounded domain with infinite ground planes.

For unbounded domains without infinite ground planes ω is the unknown value and we complete (34) by the null total charge condition¹:

$$\int_C \sigma(r) dl_C = 0, \quad (35)$$

where C is the surface of all conductors. In this case,

$$\lim_{r \rightarrow \infty} \varphi(r) = \omega. \quad (36)$$

3.2 The Numerical Algorithm

The version of boundary element method is used in the program is based on general scheme presented in [13] and [15]. The method contain mesh refinement technique [14] with enhancements [15].

Except the convenient LU direct solver for the solution of resulting linear equations with dense matrix, our program contain iterative out-of-core GMRES solver (with restarts) [16], [17]. As result, even extremely large problems can be solved with success on very poor computers using disk memory.

4 The Input File

4.1 An Example

As the example of the preparing the input data, consider the example [4], p. 906, Fig. 4b, presented on fig. (1). The input file for the example on fig. (1) is:

¹For the current version of the program, the limitation is: all conductors need to be completely embedded to a single layer of dielectric or Green function for multilayer media need to be used

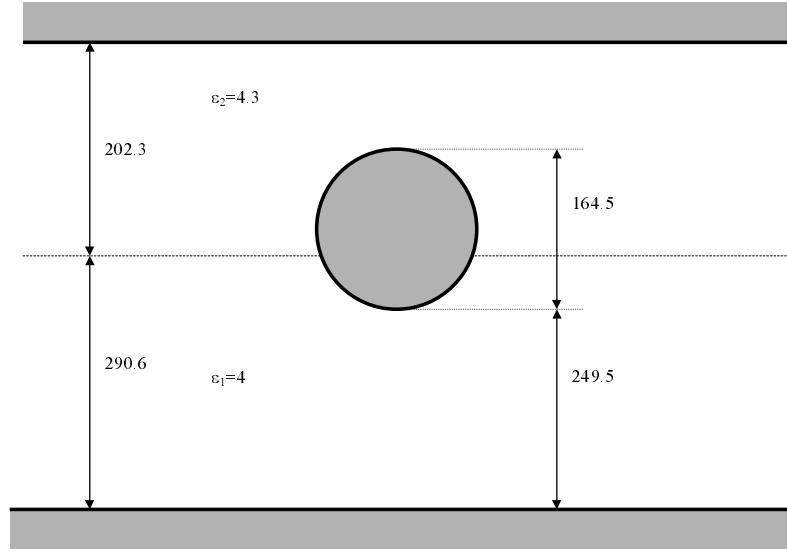


Figure 1: Single conductor with circular cross-section between two ground planes;

```

IEEE MTT-39, N6, p. 906, fig.4b, C about 210 pF/m;
Stripline configuration, demo arc elements
1 1 1 /          iprobl,iunits,ish;
4.0 4.3 /        table of deps;
4 0 492.9 /      igg[,idmn,gpars];
20.0 /          ah[,mult];
0 /             iswitch[,imm,itlim,eps];

1 1 1 1 0        / numb,iform,ittype,isign,ni[,ileft,iright];
0.0 331.75 82.25 -2.617643 -0.52394979 / g1 g2 g3 g4 g5; arc;
1 0 /            numbers of medium constants in the table;

2 1 1 1 0        / numb,iform,ittype,isign,ni[,ileft,iright];
0.0 331.75 82.25 -0.52394979 3.665542 / g1 g2 g3 g4 g5; arc;
2 0 /            numbers of medium constants in the table;

3 0 3 0 0        / numb,iform,ittype,isign,ni[,ileft,iright];
-300.0 290.6 -71.216149 290.6 / g1 g2 g3 g4; segment;
1 2 /            numbers of medium constants in the table;

4 0 3 0 0        / numb,iform,ittype,isign,ni[,ileft,iright];
71.216149 290.6 300.0 290.6 / g1 g2 g3 g4; segment;
1 2 /            numbers of medium constants in the table;

5 0 2 0 0        / numb,iform,ittype,isign,ni[,ileft,iright];
-300.0 290.6 -300.0 0.0 / g1 g2 g3 g4; segment;

```

```

0 1 /          numbers of medium constants in the table;

6 0 2 0 0      / numb,iform,ittype,isign,ni[,ileft,iright];
-300.0 492.9 -300.0 290.6 /          g1 g2 g3 g4; segment;
0 2 /          numbers of medium constants in the table;

7 0 2 0 0      / numb,iform,ittype,isign,ni[,ileft,iright];
300.0 0.0 300.0 290.6 /          g1 g2 g3 g4; segment;
0 1 /          numbers of medium constants in the table;

8 0 2 0 0      / numb,iform,ittype,isign,ni[,ileft,iright];
300.0 290.6 300.0 492.9 /          g1 g2 g3 g4; segment;
0 2 /          numbers of medium constants in the table;

```

4.2 The Input File Structure

Types of the parameters below need to be defined according to FORTRAN default conventions. The parameters with the first letters i, j, k, l, m, n are integer, the other parameters are real. Parameters in square brackets are optional and can be omitted. The structure of input file is:

```

-1st-line-of-comments-60-letters-max-----
-2nd-line-of-comments-60-letters-max-----
iprobl [iunits ish] /
eps1 [eps2 ...] /
igg [idmn gpar1 ...] /
h [mult] /
iswitch [imm itlim eps tmpdir] /
<empty string>
[-]numb iform ittype isign ni [ileft iright] / 1st line of boundary element data;
g1 g2 g3 g4 [g5 .....] / 2nd line of boundary element data;
iamns iaplus / 3rd line of boundary element data;

```

or, if -numb, then:

```

iele xsift yshift angle / 1st line of boundary element data;
iamns iaplus / 2nd line of boundary element data;
< empty string > as a border;
.....
<other boundary elements data, 3 or 2 strings for each element>
.....

```

Obligatory delimiter " / " marks end of data and beginning of comment. Each line except first two lines (comments) need to be finished by this delimiter.

The input file for the example on the fig. (1) contain the next data for the capacitance calculation of the circular in cross-section conductor between two ground planes:

- Two-ground plane Green function;
- Element 1 - lower boundary of the conductor;

- Element 2 - upper boundary of the conductor;
- Elements 3 and 4 - interface boundary;
- Elements 5,6,7,8 - artificial bounding walls (are not shown on the picture);

4.3 Input Data in General

Input data contain next parameters:

1. problem index *iprob*;
2. physical units definition *iunits* (pF/m , F/m , om/m);
3. The table of media constants *eps1*, *eps2*, ...;
4. The type of Green's function *igg*;
5. Null total charge condition switch *idmn*;
6. Green's function parameters *gpar1*, *gpar2*, ...;
7. Mesh points density parameter *h*;
8. Switch between LU and iteration solvers *iswitch*;
9. If necessary, iteration solver parameters, *tmpdir*, *imm*, *itlim*, *eps*;
10. Boundary elements (geometrical) parameters *g1*, *g2*, *g3*, *g4*, ...

As follows from (8), (22), (29) the elements of matrices C , L , G and R depend only on dimensionless, or normalized geometric data. Thus, the units of length in input data can be taken arbitrary. Nevertheless, the normalized data is recommended. In this case the probability of appearance of very small and huge numbers and roundness errors is minimal. Moreover, the values which are normalizing data units (m , μm) usually are natural for the problem.

4.4 Understanding the Boundary Elements

All boundaries in the problem (except incorporated into Green function): conductors, interfaces and walls need to be presented by set of basic curves (boundary elements). Basic curves (boundary elements) can not intersect each other. Each boundary element is carrying only one boundary condition: it is presenting or a part of a conductor, or a part of certain interface boundary, or a part of wall. In contrast with the convenient boundary element technique [12], [1], [2], [3] our boundary elements does not become smaller if the number of the degrees of freedom is increasing. The solution on each boundary element is approximated using certain refined mesh inside the program.

Inside **LCR2D**, a boundary element can be any plane, simple, smooth, oriented (plus from left side) closed or open curve. The minimal set of boundary elements is: segment, circle arc, circle.

We assume that boundary elements for conductors are oriented in the way that the body of the conductor is posed from the left side of the element. For interface boundaries, left side is arbitrary considered as "plus" (+) for the choice of the medium constants.

To define specific boundary element, input parameter is *iform* used. Certain parameters $g1, g2, \dots, i = 1 \dots N(iform)$, to set specific properties of concrete boundary element are used.

Each boundary element carry only one boundary or interface condition *itype*, *itype* = 1, 2, 3. For conductors, *itype* = 1. For walls, *itype* = 2 (zero flux condition, isolating). For interfaces, *itype* = 3.

To define the affiliation of the boundary element to a certain conductor, parameter *isign*, the number of the conductor, is used. The total number of conductors *M* is not the separate explicit parameter. The program define the total number of conductors from the input data. Numbers can not be missed.

To generate the necessary number of mesh points on each element, parameter *ni* > 0 is used. Mesh also can be generated by use of the parameter *h*.

Boundary element is also specified by it's number, *numb*. The sign of *numb* is the switch for different types of boundary element geometrical data: direct or "by reference"

From left side of each boundary element we have the permittivity ε_+ , from right side ε_- . Outside the computation domain, $\varepsilon_- = 0.0$, so orientation of the boundary element must be chosen accurately. For infinitely thin conductors both values, ε_- and ε_+ , need to be defined. The values of ε_+ and ε_- are defined from the table of *eps1, eps2, ...* by numbers *iamns, iaplus*.

4.5 Input Data Definitions

Let us remember that the types of the parameters below need to be defined according to FORTRAN default conventions. Parameters with the first letters *i, j, k, l, m, n* are integer, other parameters are real.

General parameters are:

First string Comment, of length not more 60 symbols;

Second string Comment, of length not more 60 symbols;

iprobl Type of the problem;

***iprobl* = 0** Capacitance, conductivity (non-inverted matrix);

***iprobl* = 1** Inductance, resistance (inverted matrix);

iunits Dimension units;

***iprobl* = 0, *iunits* = 0** dimensionless matrix (default);

***iprobl* = 0, *iunits* = 1** capacitance pF/m ;

***iprobl* = 0, *iunits* = 2** capacitance F/m ;

***iprobl* = 0, *iunits* = 3** conductivity $1/\Omega m$;

***iprobl* = 1, *iunits* = 0** dimensionless matrix (default);

***iprobl* = 1, *iunits* = 4** inductivity $\mu H/\mu m$;

***iprobl* = 1, *iunits* = 5** resistance Ωm ;

***iprobl* = 1, *iunits* = 6** inductance of superconductive film, μH ;

ish Switch on/off show element numbers if graphic screen;

ish = 0 off (default);

iprobl = 1 on;

eps1 eps2 ... Table of medium constants: relative permittivities for capacitance, relative permeabilities for inductance, sheet conductivities for conductivity and resistance, $\lambda_{\perp}$ for thin superconductive film;

igg Choice of the Green's function;

igg = 0 Green's function of free space $\ln(1/R)$;

igg = 1 Grounded x - axe ($y = 0$ ground plane);

igg = 2 Isolated x - axe;

igg = 3 Two-layered medium, *eps1* for upper half-plane $y > 0$, *eps2* for $y < 0$; All conductors in upper half-plane;

igg = 4 $y = 0$ and $y = gpar1$ are ground planes;

igg = 5 $y = 0$ and $y = gpar1$ are ground planes, $x = 0$ isolated;

idmn Null total charge condition switch;

idmn = 0 Do not apply this condition. Default. For bounded domain, null total charge condition is optional. It can be applied and in some cases the precision of calculations can be slightly improved;

idmn = 1 Unbounded domain, null total charge condition is placed². This option can not be used with Green functions containing infinitely stretched ground planes;

gpar1 gpar2 ... Parameters for Green's functions;

h Average mesh step for automatic mesh generation. It is necessary that $h > 0$. The mesh will be generated also by use of the parameter $ni * mult$ in the boundary element data;

iswitch Choice of linear equation solver;

iswitch = 0 LU factorization (Gauss). If there will be not enough RAM memory, iteration solver with default parameters will be implemented automatically;

iswitch = 1 Iteration solver with disk data swapping is turned on by user;

tmpdir DIR for tmp - files; Optional; Default is the current directory;

imm Internal iteration steps before restarts [16], ≥ 3 recommended. If *imm*=0 or omitted, default value (5) will be used;

eps Desired precision of iterations, if *eps* = 0.0 or omitted, default value 1.e-4 will be used;

²The limitation of the current version of the program for *idmn* = 1 is: or two-layered Green function (*igg* = 3) need to be used, or all conductors need to be embedded in the same layer of the media. In other case, results need to be verified accurately

itlim Maximum number of iterations allowed, if *itlim* = 0 or omitted, default value (50) will be used;

Boundary elements parameters:

numb The number of the boundary element is $|numb|$. If *numb* > 0, *g1*, *g2*, ... parameters define the boundary element directly. If *numb* < 0, the boundary element is defined "by reference" to other element. See the *g1*, *g2* ... definition;

iform Define the form of the element;

iform = 0 Segment;

iform = 1 Circle arc;

iform = 3 Circle;

itype Define the type of the element;

itype = 1 Conductor;

itype = 2 Wall (isolated, zero flux);

itype = 3 interface condition (drop of field, no drop of flux);

isign Define the number of the conductor, if *itype* = 1, and 0 in other case. Grounded conductor has the number 0.

ni Number of mesh points on the boundary element; There are 2 internal bounding parameters. After *ni* is read and if necessary re-calculated, it is bounded as $3 \leq ni \leq 95$;

ni > 0 The value *ni* is the number of mesh points which will be generated by the program on the boundary element.

ni = 0 Value *ni* is re-defined as the ratio of the length of the element and the parameter *h*.

ileft, irlight Mesh generation parameters.

ileft = 0 Do not refine the mesh near the origin of the element;

ileft = 1 Refine the mesh near the origin of the element (default);

irlight = 0 Do not refine the mesh near the end point of the element;

irlight = 1 Refine the mesh near the end point of the element (default);

numb > 0 **iform** = 0 4 parameters *g1*, *g2*, *g3*, *g4*. Element is the segment with origin (*g1*, *g2*) and end point (*g3*, *g4*);

iform = 1 5 parameters *g1*, *g2*, *g3*, *g4*, *g5*, arc with center (*g1*, *g2*), radius *g3*, origin angle *g4*, end point angle *g5*, both in radians;

iform = 3 4 parameters *g1*, *g2*, *g3*, *g4*, circle with center (*g1*, *g2*) and radius *g3*. If *g4* > 0 the orientation is anticlock-wise. If *g4* < 0 the orientation is clock-wise;

numb < 0 Boundary element data "by reference".

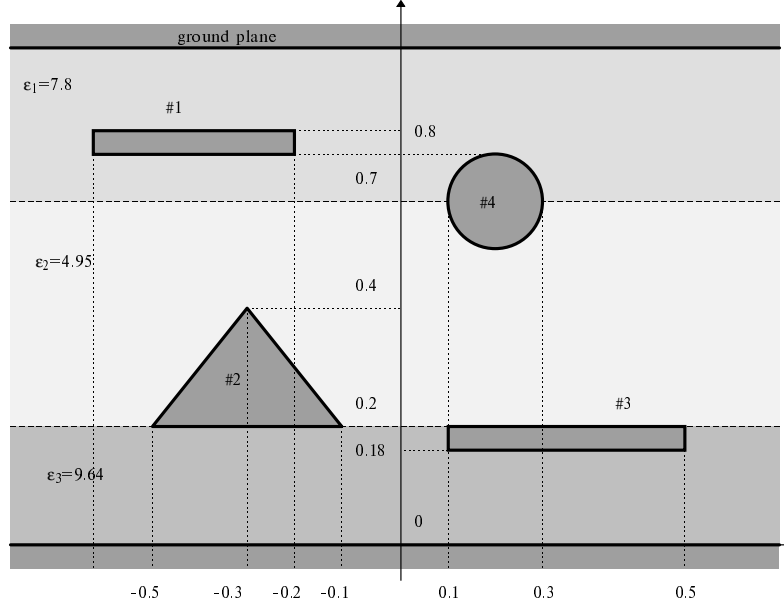


Figure 2: Four conducting transmission lines in a three-layered dielectric medium between two ground planes;

iele Reference element number. It is necessary that $|numb| > iele$;
xshift Shift in x-direction of all points of reference element;
yshift Shift in y-direction of all points of reference element;
angle Only for arcs, shift of both angles in radians;

iamns, *iaplus* The numbers of media constants (permittivity, conductivity, permeability) in the table. *iamns* is $-$ value (right side), *iaplus* is the $+$ value (left side). Predefined values are: 0 for *iamns* = 0 or *iaplus* = 0 and 1 for *iamns* = -1 or *iaplus* = -1 .

5 The Example of Calculations

Probably the most interesting test problem involving different types of conductors in multilayered media can be found in [2], p. 709, fig. 7. This configuration is presented on the fig. 2. The same problem was considered in [9] and [3]. Everywhere below, for simplicity, we consider diagonal elements of capacitance matrix only. Let us start just from number of points of discretization on each part of the boundaries taken from [2]. The total number of unknowns in this case was 84. Both ground planes were involved exactly by use of the appropriate Green function. Results, obtained by LCR2D, are appeared to approximately coincide or to be just between [2] and [9]. By densing the mesh in twice, we can estimate the rate of convergence of our algorithm. The error was not more then 0.5% (for most sparse mesh and 84 unknowns). To compare with our most

accurate results for dense mesh, the accuracy of results [2] for the triangular conductor is 7.5% and not worse 3% for other conductors. For [9], the difference is not worse 2.5% for all conductors.

The total CPU time for 84 unknowns was 2 minutes (we used PC i386/387 25MGH computer). For i486/487 120MGH PC the total CPU time is 14 seconds for 94 unknowns. These time results are comparable with calculations time [9] (6 sec for Cray XMP/48 computer).

6 Effective Calculations

The obvious way to increase the speed of calculations is to reduce the number of mesh points (degrees of freedom) in the program. In many cases it can be done without any loss of accuracy by applying special Green functions. For infinite multilayer media with strait parallel interfaces the multilayer Green function can be used. The current version of **LCR2D** contain simple two-layered Green function $igg = 3$ [5]. For more layers or for curved interfaces the direct approximation by boundary elements of the interfaces is necessary. For ground planes and walls the number of points in the mesh on the boundaries also can be reduced by applying the special Green functions. For example, if Green function for ground plane is implemented, no boundary elements for presenting this ground plane are necessary, as well as for interface boundary in two-layered Green function.

Implementation of Green function for the strip ($igg = 4, 5$) for some input data can produce numerical instability. This instability is clearly seen as oscillations of charge densities on screen graphics. The reason is in the expression for the Green function which involve exponents with large and small arguments. The exponents needs to compensate each other. For extremely small or large arguments rounding errors can appear. To avoid this problem the normalized boundary data need to be used and very dense meshes (which are really not necessary) need to be prevented.

The total number of points in the meshes also can be reduced by special choice of boundary elements and by densing the mesh on the element by use the optional parameters *ileft*, *iright* only if necessary. The control screen output with charge densities on the elements is useful in this case.

The time of calculations can be reduced for the case of the conductors embedded completely in single layer of media. In other words, the permittivity need to be the same for all points of the conductor boundary. In this case, the input data $iamns = iaplus$ for the conductor need to be applied.

7 Possible Problems and Error Messages

The program perform some checks for the consistency of the input data and print the messages for screen output and the file TRACE.LCR.

The other errors messages are the messages of FORTRAN file reading procedures. These messages means the errors in the input file. The wrong number of parameters or wrong type of parameters can be the cause of these errors.

If the end input string delimiter `"/` is omitted, input read errors can appear.

Many run-time errors can occur because of errors in preparing of boundary element data, for example, because of the intersections of boundary elements. If the intersection of some boundaries is really necessary, the elements need to be splitted by the point of the intersection. The screen output for the control of the form of the configuration under consideration is useful.

References

- [1] C. Wei, R. F. Harrington, J. R. Mautz, T. K. Sarkar, "Multiconductor transmission lines in multilayered dielectric media," *IEEE Trans. Microwave Theory Tech.*, vol. 32, pp. 439-449, April 1984.
- [2] C. Wei, R. F. Harrington, "Losses on multiconductor transmission lines in multilayered dielectric medium," *IEEE Trans. Microwave Theory Tech.*, vol. 32, pp. 705-710, July 1984.
- [3] J. Venkataraman, S. M. Rao, A. R. Djordjevic, T. K. Sarkar, Y. Naiheng, "Analysis of arbitrarily oriented microstrip transmission lines in arbitrarily shaped dielectric media over a finite ground plane," *IEEE Trans. Microwave Theory Tech.*, vol. 33 pp. 952-959, Oct. 1985.
- [4] F. Olyslager, N. Fache, D. De Zutter, "New fast and accurate line parameter calculation of general multiconductor transmission lines in multilayered media," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 901-909, June 1991.
- [5] G. L. Matthaei, G. C. Chinn, C. H. Plott, "A simplified means for computation of interconnect distributed capacitances and inductances," *IEEE Trans. Computer-Aided Design*, vol.11, pp. 574-587, June 1989.
- [6] T. Mitsuhashi, K. Yoshida, "A resistance calculation algorithm and its application to circuit extraction," *IEEE Trans. Computer-Aided Design*, vol. 6, pp. 337-345, May 1987.
- [7] Z. Wang, Q. Wu, "A two-dimensional resistance simulator using the boundary element method," *IEEE Trans. on Computer-Aided Des.*, vol. 11, pp. 497-504, April 1992.
- [8] Ruey-Beei Wu, "Resistance computations for multilayer packaging structures by applying the boundary element method," *IEEE Trans. Comp., Hybr. Manuf. Technol.*, vol. 15, N1, pp. 87-96.
- [9] S. Castillo, Z. Pantic, R. Mittra, "Finite element analysis of multiconductor printed-circuit transmission-line systems," *Int. J. Numer. Meth. Eng.*, vol. 29, pp. 1033-1047, 1990.
- [10] M. Horno, F. L. Mesa, F. Medina, R. Marques, "Quasi-TEM analysis of multilayered, multiconductor coplanar structures with dielectric and magnetic anisotropy including substrate losses," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1059-1068, August 1990.
- [11] T. F. Hayes, J. J. Barret, "Modeling of multiconductor systems for packaging and interconnecting high-speed digital IC's," *IEEE Trans. Computer-Aided Des.*, vol. 11, pp. 424-431, April 1992.
- [12] Banerjee P. K., Butterfield R., "Boundary Element Methods in Engineering Science," McGraw-Hill (UK), 1981.
- [13] Costabel M., "Principles of boundary element methods," *Computer Physics Reports*. 1987. V. 6. p. 243-274.

- [14] Y. Yan, I. H. Sloan, "Mesh grading for integral equations of the first kind with logarithmic kernel," *SIAM J. Numer. Anal.*, vol. 26, No. 3, pp 574-587, June 1989.
- [15] M. M. Khapaev (jr.), "The boundary element method for problems with corner points and mixed boundary conditions," *Differential Equations (Russian)*, 1991, vol. 27, No. 7, pp. 1263-1267.
- [16] Y. Saad, M. H. Schultz, "GMRES: A generalized minimal residual algorithm for solving non-symmetrical linear systems," *SIAM Journal on Scientific and Statistical Computing*, vol. 7, pp. 856-869, July 1986.
- [17] K. Nabors, S. Kim, J. White, "Fast capacitance extraction of general three-dimensional structures", *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 1496-1505, July 1992.