Current distribution simulation for superconducting multi-layered structures.

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Abstract. The software package 3D-MLSI which allows to calculate the current distribution and extract inductances from multi-layered high- and low-T_c superconducting circuits is developed. Both kinetic and magnetic inductances as well as 3D distribution of magnetic field are taken into account. We discuss the numerical approach used in 3D-MLSI and some new features such as visualization of sheet currents and analysis of circuits with the holes. As an example, the simulation of high-T_c double-layer transformer is presented.

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1. Motivation

Modern high- and low-T_c superconducting circuits contain components where magnetic field has essentially 3D structure. In many cases both kinetic and magnetic parts of the inductance are of the same order and equally important. These makes it necessary to develop adequate computer aided design (CAD) tools which will allow to extract the circuit parameters (e. g. inductances) from the layout. Up-to-date survey of existing software for inductances estimation in superconducting circuits can be found in [2].

Recently a new 3D technique [3, 4] and computer program 3D-MLSI for multi-layered circuits were proposed. The approach is based on London equations written for the so- called stream function instead of electric current. Finite Element Method has been used for calculations and it has proved that it is fast and precise.

In this work we consider further enhancements in the numerical technique and the 3D-MLSI software. The program allows now to calculate and view sheet currents excited by given terminal currents, by external magnetic field and by given flux trapped in the holes. Now the program can also automatically calculate the reduced inductance matrix for circuits with the holes.

To demonstrate the new features and the program efficiency we perform a simulation of high-T_c double layer YBCO transformer with the hole in the ground plane which increases the mutual inductance (transformation ratio).

2. Method

The straitforward and numerically stable method to calculate the inductances is to calculate the full energy. For this purpose it is necessary to solve a set of problems for electric current with specific excitations and for each excitation calculate full energy.

Our method assumes the thickness of superconducting films to be of same order or less than London penetration depth. Formally the method is applicable for more thick (3-5 London penetration depth) conductors. But in this case the accuracy of solution is less then in the case of thin conductors. The case of single-layer flat conductors is special because symmetry makes planarity assumptions valid for relatively thick conductors.

The approach for current solution takes into account planarity of superconducting films and is based on potential representation $\psi_m(r)$ of two-dimensional sheet current $\vec{J}_m$ which is called a *stream function*:

$$J_{m,x}(r) = \frac{\partial \psi_m(r)}{\partial y}, \quad J_{m,y}(r) = -\frac{\partial \psi_m(r)}{\partial x} \tag{1}$$

where $m = 1, \ldots, N_c$ is the number of a simple-connected conductors. Then using stationary London and Maxwell equations

$$\lambda^2 \nabla \times \vec{j} + \vec{H} = 0, \tag{2}$$

$$\nabla \times \vec{H} = \vec{j}. \tag{3}$$

where λ is London penetration depth it is possible to obtain the following expression in terms of stream functions $\psi_m(r)$:

$$-\lambda^s_m \Delta\psi_m(r_0) + \frac{1}{4\pi}\sum_{n=1}^{N_c} \iint_{S_n} (\nabla\psi_n(r), \nabla_{xy} G_{mn}(r, r_0))\, ds_r = -H_{z,ext}(r_0). \tag{4}$$

Here $\lambda^s_m = \lambda^2_m/t_m$, t_m is the thicknesses of superconducting film m and $H_{z,ext}(r_0)$ is external or exciting magnetic field.

Let $I_{h,k}$ be a full currents circulating around the hole k, then the boundary conditions for (4) are:

$$\psi_m(r) = I_{h,k},\ r \in \partial S_{h,k}, \tag{5}$$

$$\psi_m(r) = F_m(r),\ r \in \partial S_{ext,m},\ m = 1, \ldots N_c \tag{6}$$

where $S_{h,k}$ are holes boundaries and $S_{ext,m}$ are superconductor sheets boundaries. Functions $F_m(r)$ are completely defined by terminal current distribution. Moreover the kernels $G_{mn}(r, r_))$ have singularity $|r - r_0|^{-1}$. In the case of infinitely thin current sheets the kernel has the form

$$G_{mn}(r, r_0) = 1/\sqrt{|r - r_0|^2 + (h_m - h_n)^2}, \tag{7}$$

where h_k is the height of current sheet k. For the expression for superconductors of finite thickness as well as for more details concerning derivation of the equations see [3].

The main advantage of Eqs. (4), (5), and (6) is the very clear problem definition ideally suitable for Finite Element Method [6] solution. The stream function problem given by Eqs. (4), (5), and (6) is practically similar to well-known Poisson equations and boundary problems for this equation. We also want to attract the attention to the fact that excitation currents including terminal and hole circulating currents are taken into account in this approach as simple boundary conditions.

To obtain the solution, we solve Eqs. (4), (5), and (6) numerically using Finite Element Method on a triangular mesh. The solution can be used for inductance calculation or presented as color map of current density and streamlines.

3. Full Energy and Inductance Matrix

The functional of full energy of the system has the form [1, 5]:

$$E = \frac{1}{2}\sum_{n=1}^{N_c}\iint\limits_{S_n}\left(\mu_0\lambda_s^n J^2 + \vec{J}\cdot\vec{A}_s\right)ds_n =$$

$$\frac{\mu_0}{2}\sum_{n=1}^{N_c}\iint\limits_{S_n}\lambda_s^n\left(\nabla\psi_n\right)^2 ds_n +$$

$$\frac{\mu_0}{8\pi}\sum_{n=1}^{N_c}\sum_{m=1}^{N_c}\iint\limits_{S_n}ds_n\iint\limits_{S_m}\left(\nabla\psi_n,\nabla\psi_m\right)G_{mn}ds_m. \tag{8}$$

The Finite Element Method we use can be considered as procedure for minimization of full energy.

Let $N = N_h + N_t$, $\vec{I} = (I_{h,1},\dots,I_{h,N_h},I_{t,1},\dots,I_{t,N_t})$, where $I_{t,i}$ are the full currents through corresponding terminals, $i = 1,\dots,N_t$, and $I_{h,i}$ are the full currents circulating around the i-th hole, $i = 1,\dots,N_h$. Since the equations (4),(5), (6) are linear, expression (8) is a positive quadratic form with respect to $\vec{I}$. It means that there is $N\times N$ symmetric positive definite matrix L

$$E = \frac{1}{2}(L\vec{I},\vec{I}). \tag{9}$$

Matrix L is the matrix of self and mutual inductances. Fluxoids $\vec{\Phi} = (\Phi_1,\dots,\Phi_N)$ can be calculated as

$$\vec{\Phi} = L\vec{I}. \tag{10}$$

Simple proof of (10) can be found in [5].

For $i = 1,\dots,N_h$ values Φ_i in (10) are fluxoids trapped in the holes in the conductors. For $i = N_h+1,\dots,N_h+N_t$ each value Φ_i is the sum of fluxoids for the given sequence of terminals on the conductors. The fluxoids for terminal currents are discussed in [5]. In our program it is not necessary to introduce contours for calculation of these values.

In the geometries with holes, certain coefficients of the full inductance matrix can be excluded as they are sometimes useless for practical purposes. Those are the coefficients which describe the mutual and self inductances corresponding to the currents flowing around the holes. We are interested only in mutual and self inductances associated with the externally applied currents. The currents circulating around the holes are not excited independently from terminal currents. Therefore we can exclude the mutual and self inductances associated with the currents circulating around the holes (due to the trapped flux) from the inductance matrix and reduce its size. Let $\vec{\Phi} = (\vec{\Phi}^{(1)}, \vec{\Phi}^{(2)})$ where $\vec{\Phi}^{(1)} = (\Phi_1, \dots, \Phi_h)$ is the flux trapped in the holes and $\vec{\Phi}^{(2)} = (\Phi_{h+1}, \dots, \Phi_N)$ is the flux associated with the external excitation currents. Consider the same decomposition of the vector of full current I on sub-vectors $I^{(1)}$ of length h and $I^{(2)}$ of length $l = N - h$. Let us calculate the reduce inductance matrix $\tilde{L}$ of dimension $l \times l$ by excluding parts related to the hole currents $I^{(1)}$ from L. For this purpose we present the full matrix L in the block form

$$L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \tag{11}$$

where L_{11}, L_{12}, L_{21} and L_{22} are $h \times h$, $h \times l$, $l \times h$ and $l \times l$ sub-matrices of matrix L. Then, after excluding I^1 from (10) and simple calculations we obtain

$$\vec{\Phi}^{(2)} = \tilde{L} I^{(2)} + L_{21} L_{11}^{-1} \vec{\Phi}^{(1)}, \tag{12}$$

where

$$\tilde{L} = L_{22} - L_{21} L_{11}^{-1} L_{12}. \tag{13}$$

In Eq. (12) the first term contains the new reduced inductance matrix $\tilde{L}$, while the second term just says that the fluxes in the wholes affect the fluxes created by external excitation current. From Eq. (12) and (13) one can see that trapped flux $\vec{\Phi}^{(1)}$ does not enter expression for $\tilde{L}$ and, therefore, for the sake of simplicity, can be taken equal to zero. Thus, the 3D-MLSI may calculate reduced inductance matrix assuming zero trapped flux in each hole.

4. Double-layer transformer simulation

As an example of a real device, a two-layer YBCO circuit — a superconducting transformer — is calculated, see Fig. 1. Both superconducting layers are $0.2\,\mu$m thick. The first layer is a ground plane and contains $12 \times 4\,\mu$m hole to increase the mutual inductance. The second layer contains superconducting wiring. The narrow part of wires is 3 μm wide. The thickness of the insulator between the layers is 0.19 μm. London penetration depth $\lambda = 0.18\,\mu$m. The circuit is symmetric (see Fig. 1).

We assume that the ground plane carries all return currents. It is possible to calculate full 3×3 inductance matrix: for two symmetrical current loops via top layer wires and the current around the loop in the ground plane. For the circuit analysis this matrix is unsuitable because the hole circulating currents are not excited independently

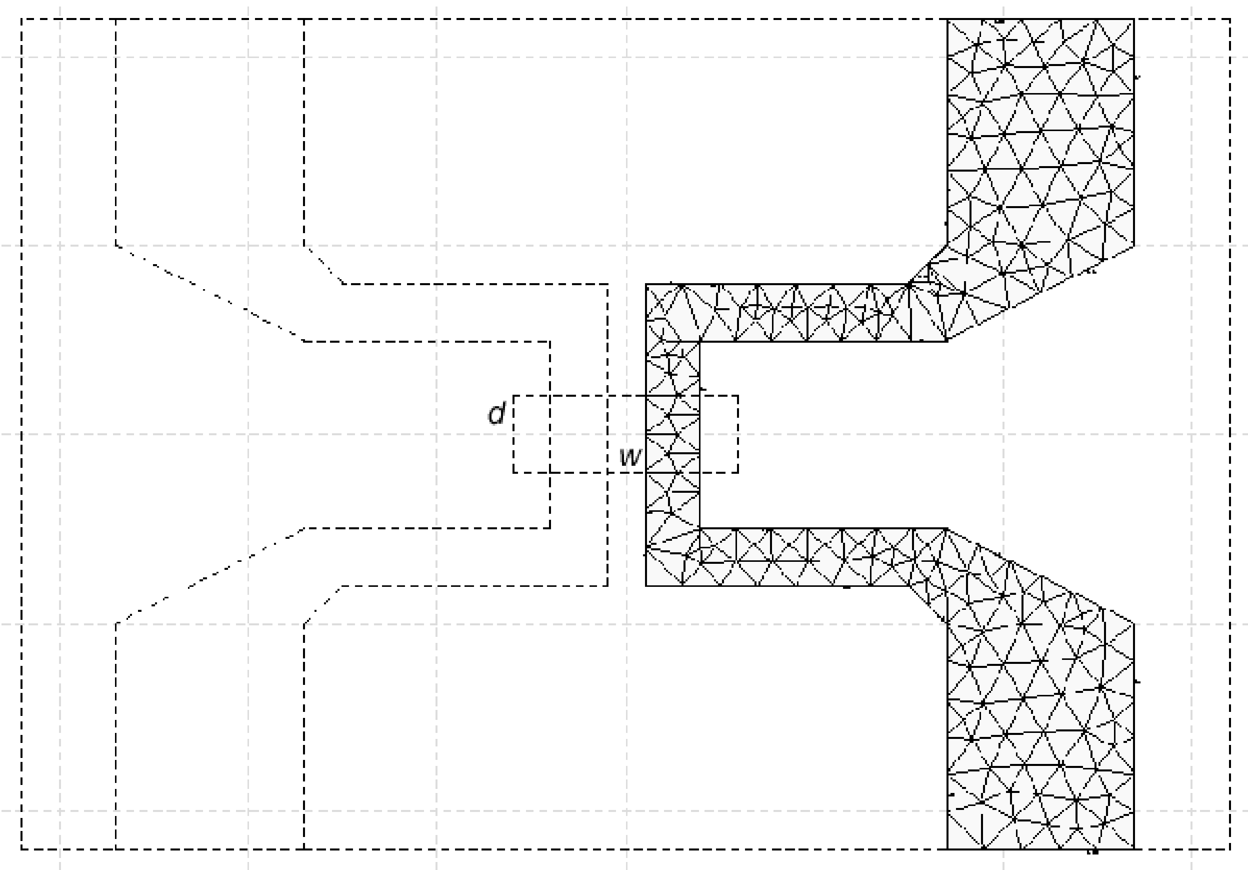

Figure 1. Transformer layout with finite element mesh on one of the conductors. Coordinate cell size 10 μm.

on the terminal currents. It is known that for any currents flowing in the circuit the hole keeps constant trapped flux, which can be taken equal to zero, as it, in any case, does not affect the reduced inductance matrix [1]. 3D-MLSI calculates 2×2 inductance matrix with self inductances L and mutual inductances $-M$. For basic configuration with rectangular hole $w \times d = 12 \times 4\,\mu$m the self-inductance is calculated to be $L = 13.61\,$pH, $M = 1.651\,$pH (transformation ratio 0.12). For the same structure without a hole $L = 10.65\,$pH, $M = 0.04\,$pH (transformation ratio 0.0039). We have also varied the hole dimensions w and d to study their influence on the transformation ratio. For these calculations we used 30×30 μm reduced central part of the layout on Fig. 1. The results are shown in Fig. 2. The plot is non-monotonous due to the curved form of wires. Designing a real circuit one can now use an optimal size of the hole which provides maximum transformation ratio. The example of current flow lines for $w = 12$ and $d = 6$ is presented in Fig. 3.

This task results in 1165 simultaneous linear equations and total CPU time 27 seconds for 1700+ Athlon computer.

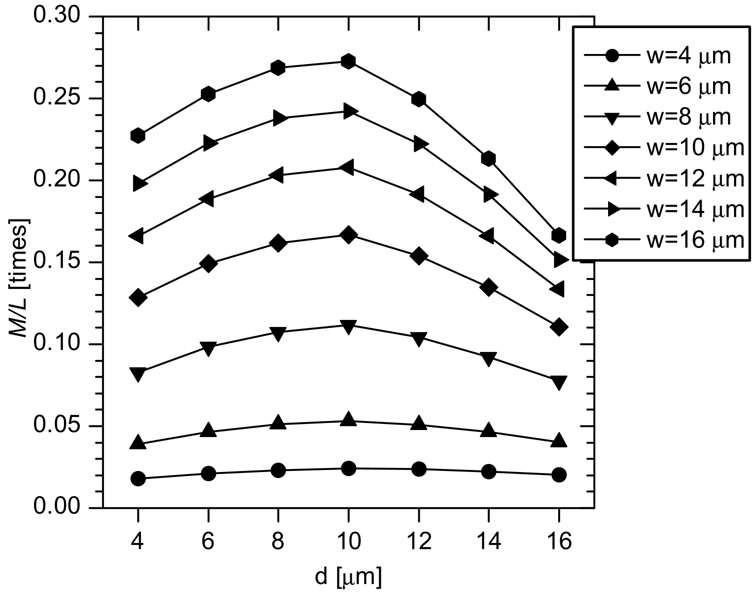

Figure 2. Transformation ratio as a function of w and d

5. Conclusions

The program 3D-MLSI proved to be the fast and reliable tool for complicated inductance calculations and for studying current distribution induced by given terminal currents, external magnetic field and magnetic flux trapped in the holes.

3D-MLSI effectively solves the problems with the large ground planes. In some modern circuits very narrow strips (in comparison with the circuit size) are used. 3D-MLSI shows good calculation speed and accuracy for such cases while other programs either loose precision or need unaffordable time for calculations.

Presently the program has some unessential limitations related to the shape of the terminals and does not allow to simulate non-planar circuits. Fortunately it is clear how to solve these problems in the future. Another improvement which we plan to implement is the phase terminals (as opposed to the current terminals used now) i.e. terminals which have a constant superconducting phase along them.

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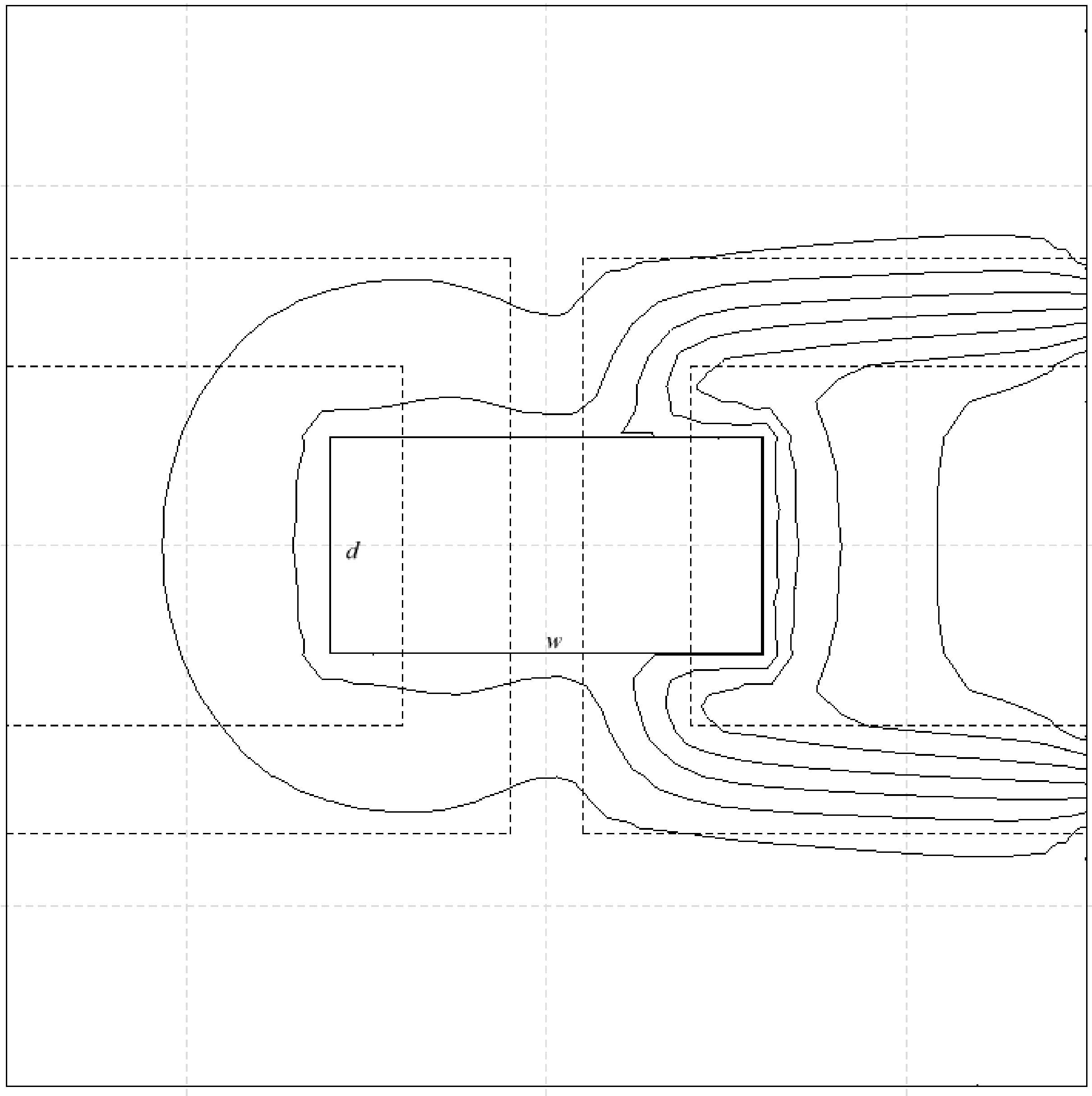

Figure 3. Current flow lines for ground plane with the hole size $w \times d = 12 \times 6\,\mu$m.

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